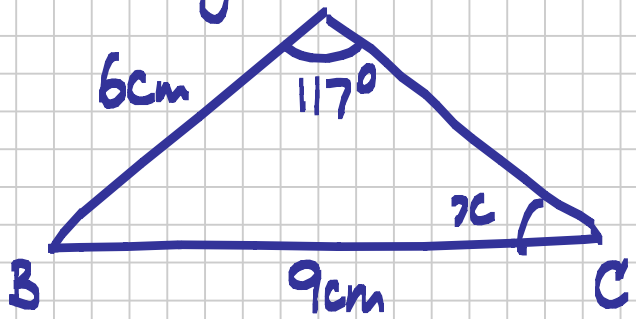
C2 Exercise 2B (sine rule used to find angles)

Note Title 21/01/2007

1 In each of the following sets of data for a triangle ABC find the value of x.

a) $AB = 6\text{cm}$, $BC = 9\text{cm}$, $\angle BAC = 117°$, $\angle ACB = x$

Draw a diagram:



$$\frac{\sin x}{6} = \frac{\sin 117}{9}$$

$$\sin x = \frac{6}{9}\sin 117$$

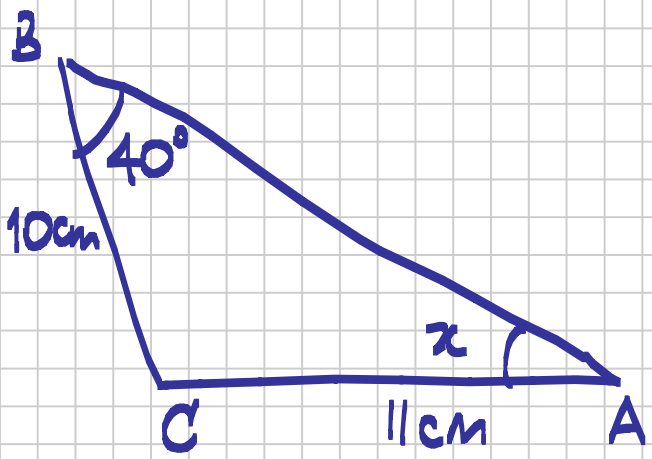
$$x = \sin^{-1}\left(\frac{2}{3}\sin 117\right)$$

$x = 36.44168693 \simeq 36.4°$

C2 Ex 2B

1 In each of the following sets of data for a triangle ABC find the value of x.

b $AC = 11\text{cm}$, $BC = 10\text{cm}$, $\angle ABC = 40°$, $\angle CAB = x$



$$\frac{\sin x}{10} = \frac{\sin 40}{11}$$

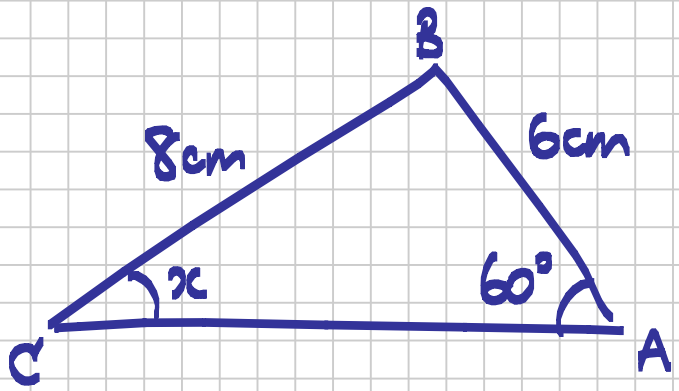
$$x = \sin^{-1}\left(\frac{10}{11}\sin 40\right)$$

$$x = 35.75725126 \simeq 35.8°$$

C2Ex2B

1 In each of the following sets of data for a triangle ABC find the value of x.

c) $AB = 6\text{cm}$, $BC = 8\text{cm}$, $\angle BAC = 60°$, $\angle ACB = x$



$$\frac{\sin x}{6} = \frac{\sin 60}{8}$$

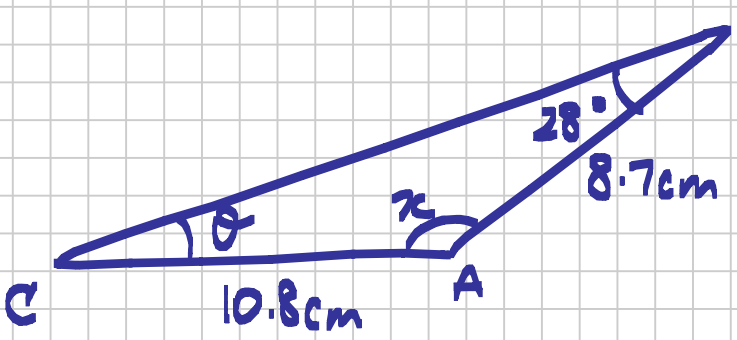
$$x = \sin^{-1}\left(\frac{3}{4}\sin 60\right)$$

$$x = 40.50535033 \simeq 40.5°$$

C2 Ex 2B

1 In each of the following sets of data for a triangle ABC find the value of x.

d) $AB = 8.7\text{cm}$, $AC = 10.8\text{cm}$, $\angle ABC = 28°$, $\angle BAC = x°$



Introduce angle $\theta = B\hat{C}A$

$$\frac{\sin\theta}{8.7} = \frac{\sin 28}{10.8}$$

$$\theta = \sin^{-1}\left(\frac{8.7}{10.8}\sin 28\right) = 22.22132895°$$

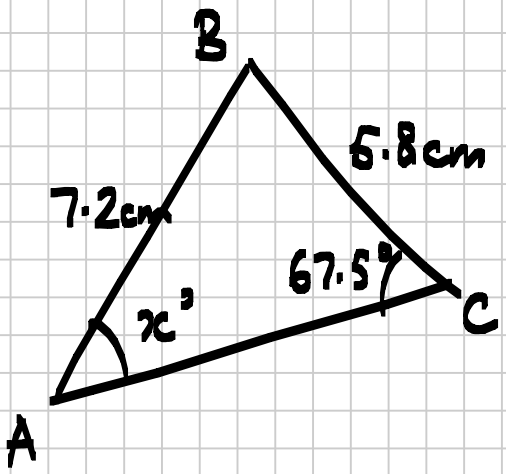
$$x = 180 - (\theta + 28) = 180 - 50.2213\ldots = 129.7786711$$

so $x \simeq 130°$.

C2 Ex 2B

2 In each of the diagrams work out the value of x.

a)



$$\frac{\sin x}{5.8} = \frac{\sin 67.5}{7.2}$$

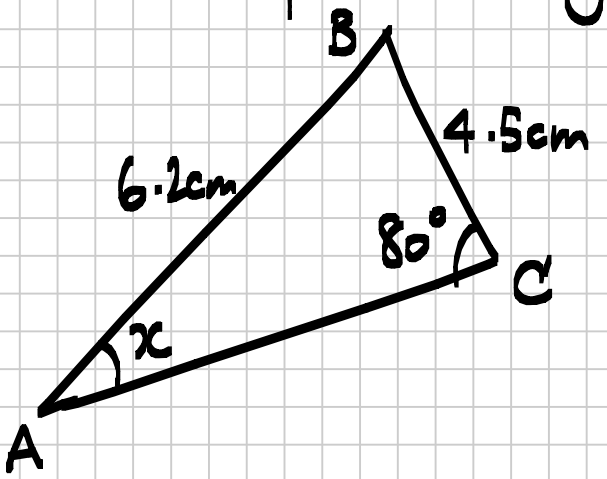
$$x = \sin^{-1}\left(\frac{5.8}{7.2}\sin 67.5\right)$$

$$x = 48.09354391 \approx 48.1°$$

C2 Ex 2B

2 In each of the diagrams work out the value of x.

b)



$$\frac{\sin 80}{6.2} = \frac{\sin x}{4.5}$$

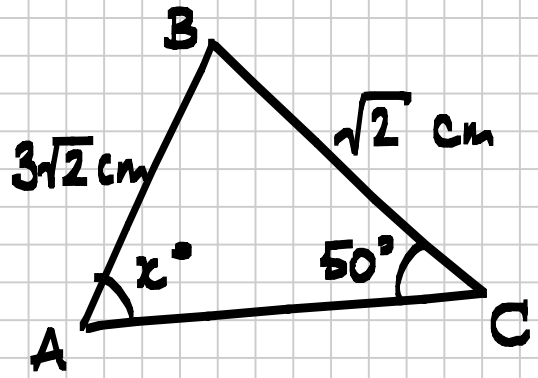
$$x = \sin^{-1}\left(\frac{4.5}{6.2}\sin 80^\circ\right)$$

$$x = 45.625\,15758 \approx 45.6^\circ$$

C2 Ex 2B

2 In each of the diagrams work out the value of x.

c)



$$\frac{\sin x}{\sqrt{2}} = \frac{\sin 50}{3\sqrt{2}}$$

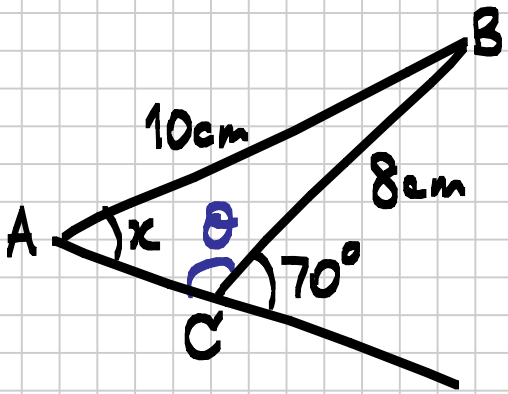
$$x = \sin^{-1}\left(\frac{1}{3}\sin 50°\right)$$

$$x = 14.79421551 \simeq 14.8°$$

C2 Ex 2B

2 In each of the diagrams work out the value of x.

d)



Call $A\hat{C}B = \theta = 110°$ (angles on straight line)

$$\frac{\sin x}{8} = \frac{\sin\theta}{10}$$

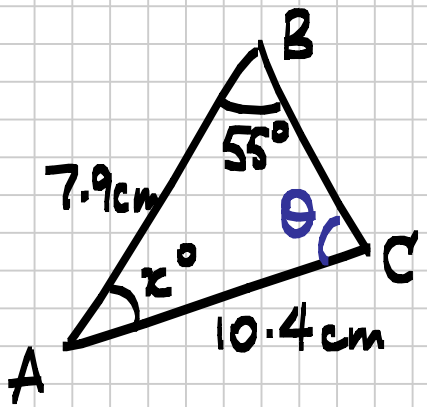
$$x = \sin^{-1}\left(\frac{4}{5}\sin 110°\right)$$

$$x = 48.74255246 \simeq 48.7°$$

C2 Ex 2B

2 In each of the diagrams work out the value of x.

e)



Call $ACB = \theta$ then $x = 180 - (55 + \theta)$ later.

$$\frac{\sin\theta}{7.9} = \frac{\sin 55}{10.4}$$

$$\theta = \sin^{-1}\left(\frac{7.9}{10.4}\sin 55\right)$$

$$\theta = 38.47993254$$

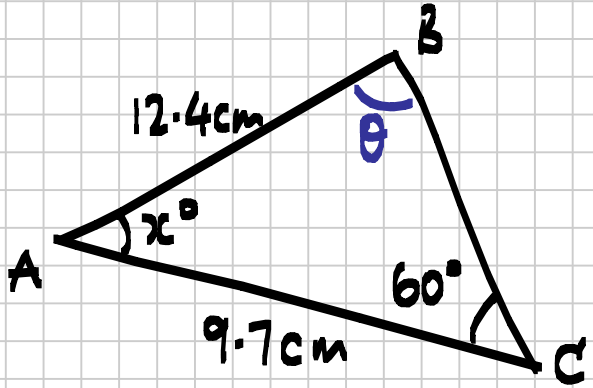
$$x = 180 - 93.4799\ldots = 86.5200674\text{6}$$

so $x \simeq 86.5°$

C2 Ex 2B

2 In each of the diagrams work out the value of x.

f)



Introduce $\theta = A\hat{B}C$

$$\frac{\sin\theta}{9.7} = \frac{\sin 60}{12.4}$$

$$\theta = \sin^{-1}\left(\frac{9.7}{12.4}\sin 60°\right)$$

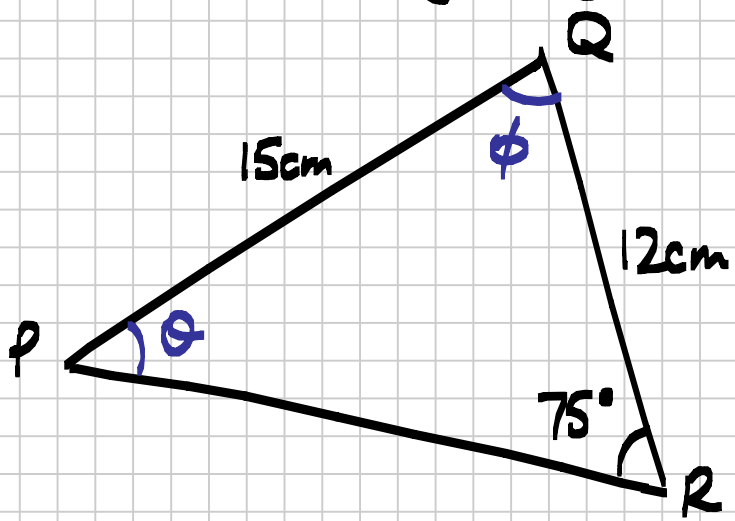
$$\theta = 42.645114\,19°$$

$$x = 180 - 60 - \theta = 77.354\,8858\,1$$

$$x \approx 77.4°$$

C2 Ex2B

3 In $\triangle PQR$, $PQ = 15\text{cm}$, $QR = 12\text{cm}$ and $\angle PRQ = 75°$. Find the two remaining angles



Call them $\theta = R\hat{P}Q$ and $\phi = P\hat{Q}R$

Then $\frac{\sin\theta}{12} = \frac{\sin 75}{15}$

$\theta = \sin^{-1}\left(\frac{4}{5}\sin 75^{*}\right)$

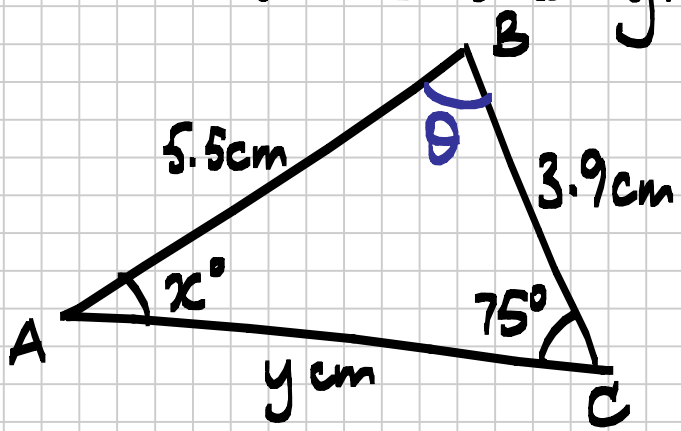
$\theta = 50.600639\,73° \simeq 50.6°$

Since $\phi = 180 - 75 - \theta = 105 - \theta$, $\phi = 54.399360\,22 \simeq 54.4°$

* you may recall we found $\sin 75 = \frac{\sqrt{2}(1+\sqrt{3})}{4}$ from C2 Ex2A Q3

C2 Ex 2B

4a... work out x and y.



$$\frac{\sin 75}{5.5} = \frac{\sin x}{3.9}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{3.9}{5.5}\sin 75°\right)$$

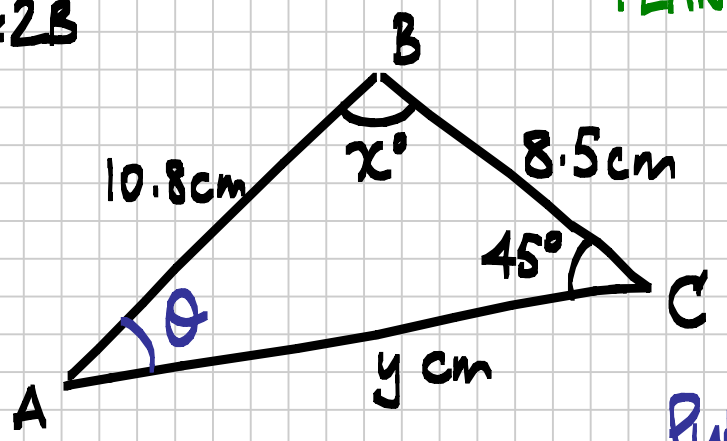
$$\Rightarrow x = 43.23004114 \simeq 43.2°$$

$\angle ABC = \theta = 61.8°$. now $\quad \frac{y}{\sin\theta} = \frac{5.5}{\sin 75}$

so $\quad y = \frac{5.5\sin\theta}{\sin 75} = 5.016747118 \simeq 5.02\text{cm}$

C2 Ex2B

4b



PLAN: Introduce angle θ, you can't work out x and y at the same time.
Find θ first & use angle sum of a triangle to get x then sine rule again to find y.

Put $\theta = B\hat{A}C$ then using the sine rule

$$\frac{\sin\theta}{8.5} = \frac{\sin 45}{10.8} \Rightarrow \theta = \sin^{-1}\left(\frac{8.5}{10.8}\sin 45^\circ\right)$$

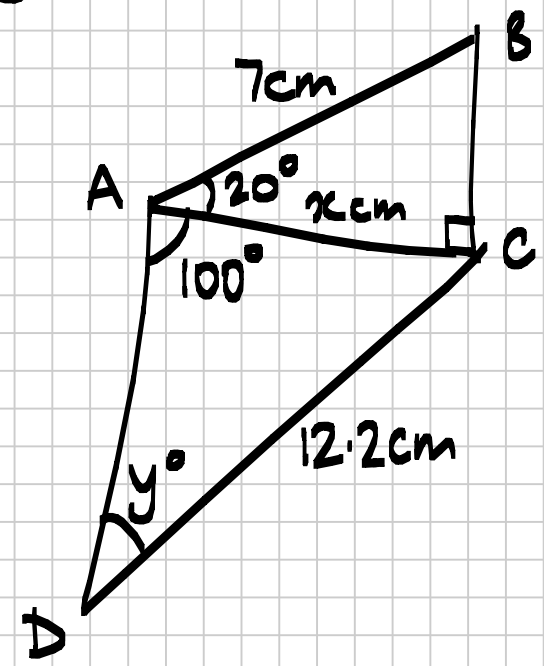
$$\Rightarrow \theta = 33.81541917 \simeq 33.8^\circ \Rightarrow x = 180-(45+\theta) \simeq 101^\circ$$

Now $$\frac{10.8}{\sin 45} = \frac{y}{\sin x} \Rightarrow y = \frac{10.8\sin x}{\sin 45} = 14.98342274$$

so $y \simeq 15.0$ cm

C2 Ex 2B

4c



Note $\triangle ABC$ is a right angled triangle

So $\cos 20^\circ = \frac{x}{7}$ using trigonometry.

$\Rightarrow x = 7\cos 20$

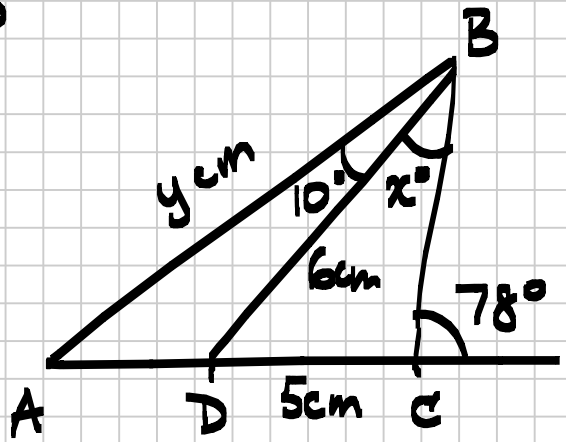
$x = 6.577848346 \simeq 6.58\text{cm}$

$$\frac{\sin y}{x} = \frac{\sin 100}{12.2}$$

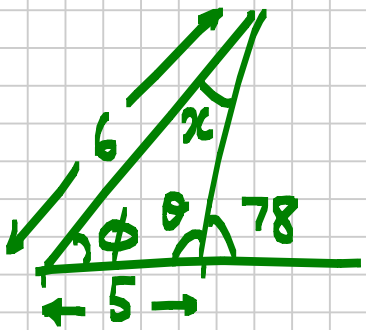
$$\Rightarrow y = \sin^{-1}\left(\frac{x}{12.2}\sin 100\right)$$

$$\Rightarrow y = 32.071472 \simeq 32.1^\circ$$

C2 Ex 2B
4d

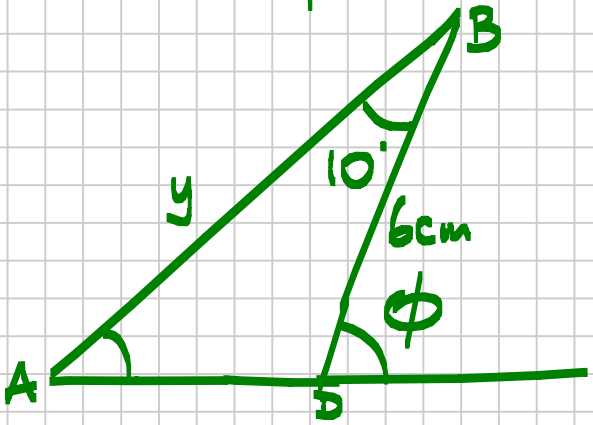


Plan: ① concentrate on this bit:



find θ (angles on line)
then x (sine rule)

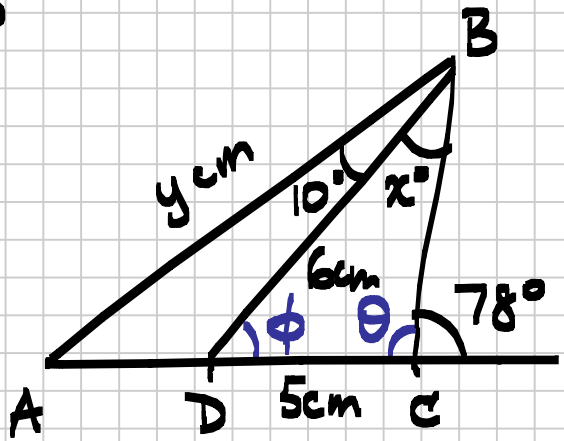
② Next find ϕ (angles in Δ) and start to concentrate on the left hand Δ:



Then you can find angles $A\hat{D}B$ and $B\hat{A}D$ to use sine rule again.

(Quite a hard question)

C2 Ex 2B
4d



① $\theta = 180 - 78$ (angles on straight line)
$\Rightarrow \theta = 102°$

$$\frac{\sin x}{5} = \frac{\sin\theta}{6} \Rightarrow x = \sin^{-1}\left(\frac{5}{6}\sin\theta\right)$$

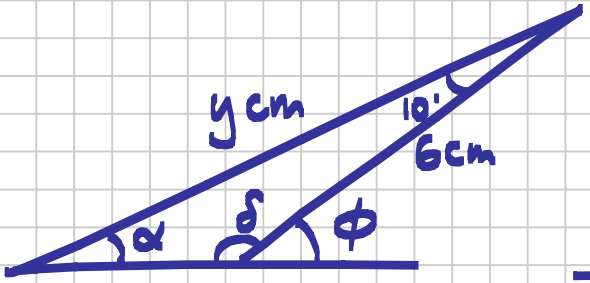
$x \approx 54.6°$

② Angle sum of $\Delta \Rightarrow \phi \approx 23.4$

$\delta = 180 - \phi \approx 157$

$\alpha = 180 - (\delta + 10) = 180 - (180 - \phi + 10)$

$\Rightarrow \alpha = \phi - 10 \approx 13.4$

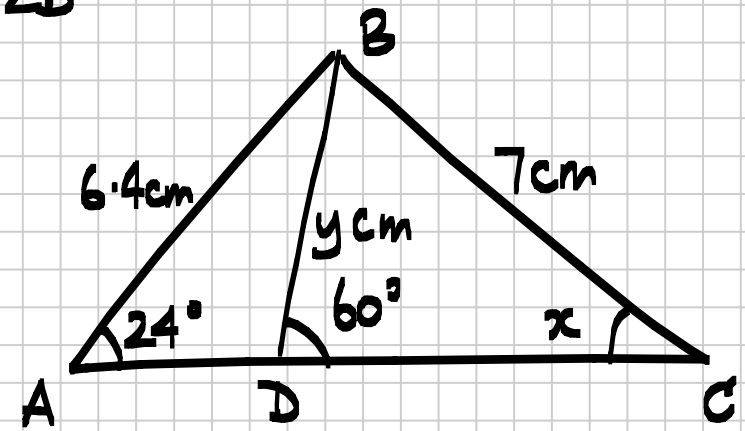


$$\frac{y}{\sin\delta} = \frac{6}{\sin\alpha} \Rightarrow y = \frac{6\sin\delta}{\sin\alpha}$$

so $y = \dfrac{6\sin 157}{\sin 13.4} = 10.28207848 \approx 10.3$ cm

C2 Ex 2B

4 e



$$\frac{\sin x}{6.4} = \frac{\sin 24}{7} \quad \text{in } \Delta ABC$$

$$\Rightarrow x = \sin^{-1}\left(\frac{6.4}{7}\sin 24\right)$$

$$x = 21.83120775 \simeq 21.8°$$

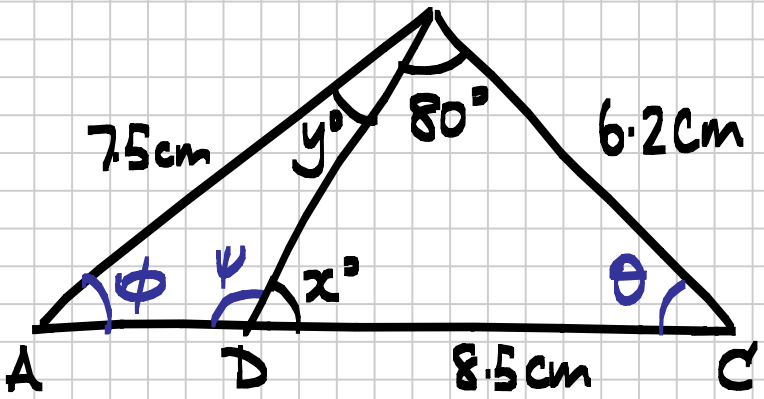
Then in ΔDBC

$$\frac{y}{\sin x} = \frac{7}{\sin 60} \Rightarrow y = \frac{7\sin x}{\sin 60}$$

$$y = 3.005817733 \simeq 3.01 \text{ cm}$$

C2 Ex 2B
4f

θ = theta ϕ = phi ψ = chi ω = omega



$$\frac{\sin x}{6.2} = \frac{\sin 80^\circ}{8.5}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{6.2}{8.5}\sin 80^\circ\right)$$

$$x = 45.91680345 \simeq 45.9^\circ$$

$\psi = 180 - x \simeq 134$

$\theta = 180 - (80 + x) \simeq 54.1^\circ$

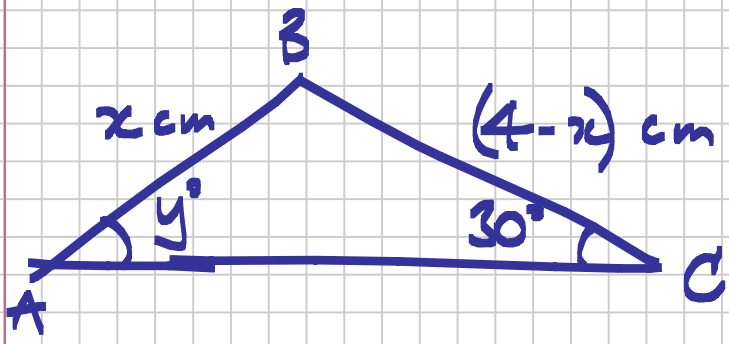
$$\frac{\sin\phi}{6.2} = \frac{\sin\theta}{7.5} \Rightarrow \phi = \sin^{-1}\left(\frac{6.2}{7.5}\sin\theta\right) \simeq 42.0^\circ$$

$\phi + \psi + y = 180 \Rightarrow y = 180 - (\phi + \psi) = 3.888916235 \simeq 3.89^\circ$ *

* there appears to be an early rounding error in book which gives 3.87° instead. I've calculated using memory functions & carried variables.

C2 Ex2B

5 In $\triangle ABC$, $AB = x$ cm, $BC = (4-x)$ cm, $\angle BAC = y°$ and $\angle BCA = 30°$. Given that $\sin y = \frac{1}{\sqrt{2}}$ show that $x = 4(\sqrt{2}-1)$



Using the sine rule:

$$\frac{4-x}{\sin y} = \frac{x}{\sin 30}$$

$$\Rightarrow (4-x)(\sin 30)^{*} = x \sin y$$

$$\Rightarrow \frac{1}{2}(4-x) = \frac{1}{\sqrt{2}}x \Rightarrow \sqrt{2}(4-x) = 2x$$

$$\Rightarrow 4\sqrt{2} - \sqrt{2}x = 2x$$

$$\Rightarrow 4\sqrt{2} = 2x + \sqrt{2}x = (2+\sqrt{2})x$$

* You should know $\sin 30° = 1/2$ using half an equilateral triangle. See next page...

We have $4\sqrt{2} = (2+\sqrt{2})x$

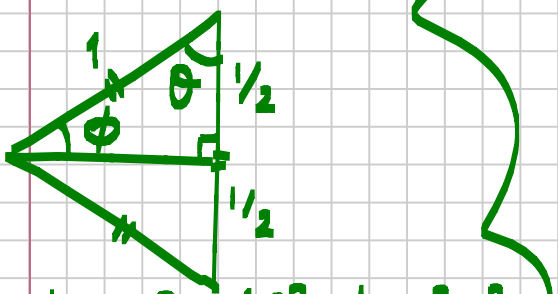
so $x = \dfrac{4\sqrt{2}}{2+\sqrt{2}} \times \dfrac{2-\sqrt{2}}{2-\sqrt{2}}$

$x = \dfrac{8\sqrt{2}-8}{4-2}$

$x = \dfrac{8(\sqrt{2}-1)}{2}$

$x = 4(\sqrt{2}-1)$

Consider the equilateral triangle of side length 1:



Realise $\theta = 60°$, $\phi = 30°$
$\Rightarrow \sin 30 = \cos 60 = \frac{1}{2}$.